# Galactic Parameters from Masers with Trigonometric Parallaxes 

V. Bobylev, A. Bajkova, and A. Stepanishchev

Pulkovo Observatory, St. Petersburg, Russia

June 2010

## INTRODUCTION



Fig. 2.- Peculiar motion vectors of high mass star forming regions (superposed on an artist conception) projected on the Galactic plane after transforming to a reference frame rotating with the Galaxy, using IAU standard values of $R_{0}=8.5 \mathrm{kpc}$ and $\Theta_{0}=220 \mathrm{~km} \mathrm{~s}^{-1}$ and a flat rotation curve. A $10 \mathrm{~km} \mathrm{~s}^{-1}$ motion scale is in the lower left. The Galaxy is viewed from the north Galactic pole and rotates clockwise.

1. M.J. Reid, K.M. Menten, X.W. Zheng, et al., 2009, used NRAO (VLBA) and Japanese VERA project
to measure trigonometric parallaxes and proper motions of masers. Results from 18 sources are:

Distance to the Galactic center

$$
\text { R0 = } 8.4 \text { +- } 0.6 \text { kpc; }
$$

Circular rotation speed
$\mathrm{V} 0=254+-16 \mathrm{~km} / \mathrm{s} ;$
$\mathrm{dV} / \mathrm{dR}=2.3+-9 \mathrm{~km} / \mathrm{s} / \mathrm{kpc}$.
Angular velocity of galactic rotation is
$\mathrm{V} 0 / \mathrm{Ro}=30.3+-0.9 \mathrm{~km} / \mathrm{s} / \mathrm{kpc}$.
They found that star forming regions on average are orbiting the Galaxy $\approx 15 \mathrm{~km} / \mathrm{s}$ slower than expected for circular orbits.
2. Analysis of motions of 18 masers was made by a number of authors - Reid et al., (2009); Baba et al., (2009); Bovy et al., (2009); McMillan \& Binney, (2010).

Another problem is uncertainty of a peculiar velocity of the Sun with respect to the Local Standart of Rest (LSR)

Reid et al., (2009) lag $\approx 15 \mathrm{~km} / \mathrm{s}$ is based on $(\mathrm{U}, \mathrm{V}, \mathrm{W}) L S R=(10,5,7) \mathrm{km} / \mathrm{s}$ determined by Dehnen \& Binney (1998).

But recently in the work by Schonrich et al. (2009), where gradient of metallicity of stars in the Galactic disk was taken into account, this velocity is different: $(\mathrm{U}, \mathrm{V}, \mathrm{W}) \mathrm{LSR}=(11.1,12.2,7.3) \pm(0.7,0.5,0.4) \mathrm{km} / \mathrm{s}$.

McMillan \& Binney (2010) suggested that the value of VLSR component should be increased from $5 \mathrm{~km} / \mathrm{s}$ to $11 \mathrm{~km} / \mathrm{s}$.

## AIMS

We are trying to establish relationship between motions of all currently known masers (28) having parallaxes, proper motions and line-of-sight velocities, and parameters of the Galactic spiral density waves, and to estimate non-perturbed components of the peculiar velocity of the Sun with respect to the LSR.

This goal is achieved by determining parameters of the Galactic rotation curve, as well as other kinematic parameters, by means of Bottlinger's equations.

Fourier analysis of periodic deviations of circular velocity from the Galactic rotation curve found and of galactocentric radial velocities of masers allows us to obtain some estimates of spiral density wave parameters.

Table 1. Data on masers. Velocities $U^{1}, V^{1}, W^{1}$ are residual velocities with Galactic rotation excluded using parameters (17).

| Source | $R, \mathrm{kpc}$ | $U^{1}, \mathrm{~km} \mathrm{~s}^{-1}$ | $V^{1}, \mathrm{~km} \mathrm{~s}^{-1}$ | $W^{1}, \mathrm{~km} \mathrm{~s}^{-1}$ | $\Delta V_{\theta}, \mathrm{km} \mathrm{s}^{-1}$ | $V_{R}, \mathrm{~km} \mathrm{~s}^{-1}$ | Ref |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| L 1287 | 8.52 | $-2.9 \pm 1.7$ | $-21.6 \pm 2.6$ | $-10.2 \pm 2.5$ | $-5.2 \pm 2.6$ | $-5.8 \pm 1.7$ | $(2)$ |
| IRAS 00420+5530 | 9.33 | $2.8 \pm 2.7$ | $-22.2 \pm 4.2$ | $-1.5 \pm 0.8$ | $-4.0 \pm 4.2$ | $-12.0 \pm 2.8$ | $(1)$ |
| NGC 281-W | 9.50 | $-4.5 \pm 2.6$ | $-5.3 \pm 2.8$ | $-16.0 \pm 1.7$ | $11.2 \pm 2.8$ | $-1.4 \pm 2.6$ | $(2)$ |
| W3 (OH) | 9.46 | $4.5 \pm 2.1$ | $-20.5 \pm 2.2$ | $-6.2 \pm 0.2$ | $-2.7 \pm 2.2$ | $-13.3 \pm 2.1$ | $(1)$ |
| WB 89-437 | 13.05 | $0.4 \pm 2.1$ | $-17.8 \pm 2.1$ | $2.5 \pm 3.8$ | $1.0 \pm 2.1$ | $-8.7 \pm 2.1$ | $(1)$ |
| NGC 1333 (f1) | 8.21 | $-23.6 \pm 4.4$ | $-13.8 \pm 2.3$ | $0.0 \pm 2.5$ | $2.0 \pm 2.3$ | $15.4 \pm 4.4$ | $(3)$ |
| NGC 1333 (f2) | 8.21 | $-21.1 \pm 4.5$ | $5.2 \pm 1.8$ | $10.0 \pm 3.5$ | $21.0 \pm 1.8$ | $13.1 \pm 4.5$ | $(3)$ |
| Ori GMR A | 8.32 | $-4.8 \pm 4.1$ | $-9.3 \pm 2.3$ | $-3.1 \pm 1.7$ | $6.5 \pm 2.3$ | $-3.6 \pm 4.1$ | $(5)$ |
| Ori KL | 8.34 | $-18.0 \pm 4.2$ | $-14.9 \pm 2.4$ | $-3.8 \pm 3.0$ | $1.2 \pm 2.4$ | $9.8 \pm 4.2$ | $(1)$ |
| S252 A | 10.08 | $-12.5 \pm 3.0$ | $-15.5 \pm 0.6$ | $-9.2 \pm 0.3$ | $0.6 \pm 0.6$ | $4.3 \pm 3.0$ | $(1)$ |
| S255 | 9.56 | $-7.7 \pm 3.6$ | $-3.9 \pm 9.5$ | $-4.0 \pm 10$ | $11.9 \pm 9.5$ | $-1.0 \pm 3.6$ | $(2)$ |
| S269 | 13.16 | $-3.7 \pm 2.9$ | $-16.1 \pm 1.2$ | $-11.7 \pm 0.9$ | $-0.7 \pm 1.2$ | $-4.4 \pm 2.9$ | $(1)$ |
| VY CMa | 8.64 | $-6.1 \pm 2.9$ | $-18.0 \pm 3.0$ | $-13.6 \pm 2.5$ | $-2.4 \pm 3.0$ | $-1.8 \pm 2.9$ | $(1)$ |
| G232.62+0.9 | 9.12 | $-7.7 \pm 3.2$ | $-12.6 \pm 3.1$ | $-6.4 \pm 2.2$ | $3.2 \pm 3.1$ | $-1.0 \pm 3.2$ | $(1)$ |
| G9.62+0.20 | 3.04 | $-59.3 \pm 2.5$ | $-7.4 \pm 9.4$ | $-17.4 \pm 2.9$ | $-6.4 \pm 9.1$ | $51.4 \pm 3.6$ | $(4)$ |
| G14.33-0.64 | 6.92 | $0 \pm 10$ | $-9 \pm 14$ | $-10.8 \pm 4.5$ | $7 \pm 14$ | $-8 \pm 10$ | $(1)$ |
| G23.66-0.13 | 5.23 | $21.5 \pm 3.7$ | $-8.3 \pm 5.7$ | $-3.1 \pm 0.4$ | $14.6 \pm 5.6$ | $-26.9 \pm 3.8$ | $(1)$ |
| G23.01-0.41 | 4.18 | $-0.4 \pm 4.7$ | $-25.1 \pm 9.1$ | $-8.5 \pm 2.3$ | $-5.0 \pm 8.4$ | $-11.0 \pm 5.8$ | $(1)$ |
| G23.43-0.18 | 3.50 | $-18 \pm 10$ | $3 \pm 22$ | $-5.3 \pm 1.8$ | $7 \pm 18$ | $20 \pm 16$ | $(1)$ |
| G35.20-0.74 | 6.34 | $-21.3 \pm 3.3$ | $-19.8 \pm 3.6$ | $-15.7 \pm 1.5$ | $-6.5 \pm 3.6$ | $12.1 \pm 3.3$ | $(1)$ |
| W48 | 5.65 | $-30.9 \pm 5.7$ | $-17.3 \pm 7.5$ | $-16.5 \pm 2.4$ | $-8.9 \pm 7.3$ | $21.0 \pm 5.9$ | $(1)$ |
| W51 | 6.08 | $-24 \pm 41$ | $-11 \pm 35$ | $-10.4 \pm 3.7$ | $-6 \pm 37$ | $15 \pm 38$ | $(1)$ |
| V645 | 7.16 | $-13.8 \pm 2.6$ | $-13.1 \pm 2.9$ | $-11.4 \pm 0.5$ | $1.3 \pm 2.9$ | $6.2 \pm 2.6$ | $(1)$ |
| Onsala1 | 7.50 | $-9 \pm 13$ | $-17.9 \pm 5.6$ | $-2.9 \pm 6.0$ | $-2.1 \pm 6.8$ | $0 \pm 12$ | $(2)$ |
| IRAS 22198 | 8.26 | $-4.1 \pm 3.7$ | $-23.8 \pm 4.9$ | $3.0 \pm 2.1$ | $-7.5 \pm 4.9$ | $-4.8 \pm 3.7$ | $(3)$ |
| L 1206 | 8.27 | $-12.7 \pm 4.4$ | $-19.8 \pm 3.2$ | $-7.2 \pm 5.7$ | $-4.3 \pm 3.2$ | $4.2 \pm 4.3$ | $(2)$ |
| Cep A | 8.26 | $-8.8 \pm 3.8$ | $-17.0 \pm 4.9$ | $-12.6 \pm 1.2$ | $-1.2 \pm 4.8$ | $0.5 \pm 3.8$ | $(1)$ |
| NGC 7538 | 9.30 | $-2.4 \pm 2.1$ | $-35.2 \pm 2.9$ | $-18.1 \pm 1.0$ | $-17.1 \pm 2.8$ | $-10.7 \pm 2.2$ | $(1)$ |
| S Per | 9.30 | $-16.6 \pm 1.2$ | $-20.9 \pm 1.2$ | $-17.2 \pm 3.1$ | $-6.5 \pm 1.2$ | $7.4 \pm 1.2$ | $(6)$ |

Notes. Initial data are taken from: (1) - Reid et al. (2009); (2) - Rygl et al. (2009); (3) - Hirota et al. (2008a); (4) - Sanna et al. (2010); (5) - Sandstrom et al. (2007); (6) - Asaki et al. (2007).


Figure 1. Coordinates of masers in the $X Y$ Galactic plane. Locations of the Sun and Galactic Centre (GC) are indicated. The circle of 8 kpc radius is drawn.


### 3.2 Determining Rotation Curve Parameters

Method used here is based on the well-known Bottlinger's formulas (Ogorodnikov, 1958), where angular velocity of Galactic rotation is expanded in a series to $n$-th order terms in $r / R_{0}$ :
$V_{r}=-u_{\odot} \cos b \cos l-v_{\odot} \cos b \sin l-w_{\odot} \sin b-$
$-R_{0} \sin l \cos b\left[\left(R-R_{0}\right) \Omega_{0}^{1} / 1!+\ldots+\left(R-R_{0}\right)^{n} \Omega_{0}^{n} / n!\right]$,
$V_{l}=u_{\odot} \sin l-v_{\odot} \cos l+r \Omega_{0} \cos b-$
$-\left(R_{0} \cos l-r \cos b\right)\left[\left(R-R_{0}\right) \Omega_{0}^{1} / 1!+\right.$
$\left.+\ldots+\left(R-R_{0}\right)^{n} \Omega_{0}^{n} / n!\right]$,
$V_{b}=u_{\odot} \cos l \sin b+v_{\odot} \sin l \sin b-w_{\odot} \cos b+$
$+R_{0} \sin l \sin b\left[\left(R-R_{0}\right) \Omega_{0}^{1} / 1!+\ldots+\left(R-R_{0}\right)^{n} \Omega_{0}^{n} / n!\right]$,
where $V_{r}$ is heliocentric line-of-sight velocity; $V_{l}=$ $4.74 r \mu_{l} \cos b$ and $V_{b}=4.74 r \mu_{b}$ are proper motion velocity components in the $l$ and $b$ directions, respectively (the coefficient 4.74 is the quotient of the number of kilometers in astronomical unit by the number of seconds in a tropical year) $; r=1 / \pi$ is the heliocentric distance of an object; proper motion components $\mu_{l} \cos b$ and $\mu_{b}$ are in mas $\mathrm{yr}^{-1}$, line-of-sight velocity $V_{r}$ is in $\mathrm{km} \mathrm{s}^{-1} ; u_{\odot}, v_{\odot}, w_{\odot}$ are Solar velocity components with respect to the mean group velocity under consideration; $R_{0}$ is the galactocentric distance of the Sun; $R$ is the galactocentric distance of an object; $R, R_{0}$



Figure 2. Galactic rotation curves: (a) $\Omega(R)$ and (b) $V_{\theta}(R)$ for adopted $R_{0}=7.5,8.0,8.5 \mathrm{kpc}$, lines 1,2 and 3 .

Then we obtained solution of (1)-(3) for the fixed value of $R_{0}=8.0 \mathrm{kpc}$ for 84 equations. As a result we found the following parameters of the Solar velocity with respect to the group velocity: $\left(u_{\odot}, v_{\odot}, w_{\odot}\right)=(8.0,14.9,7.6) \pm$ $(2.1,1.9,1.6) \mathrm{km} \mathrm{s}^{-1}$, and parameters of the angular velocity of Galactic rotation:

$$
\begin{gather*}
\Omega_{0}=-31.0 \pm 1.2 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{kpc}^{-1}  \tag{17}\\
\Omega_{0}^{\prime}=+4.46 \pm 0.21 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{kpc}^{-2} \\
\Omega_{0}^{\prime \prime}=-0.876 \pm 0.067 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{kpc}^{-3}
\end{gather*}
$$

The unit weight error is $\sigma_{0}=9 \mathrm{~km} \mathrm{~s}^{-1}$. Oort constants $A=0.5 R_{0} \Omega_{0}^{1}, B=\Omega_{0}+0.5 R_{0} \Omega_{0}^{1}$ are: $A=17.8 \pm$ $0.8 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{kpc}^{-1}$ and $B=-13.2 \pm 1.5 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{kpc}^{-1}$. Using these values and taking into account the error $\sigma_{R_{0}}=0.3 \mathrm{kpc}$, we estimated the circular velocity of the Solar neighborhood $V_{0}=\left|R_{0} \Omega_{0}\right|=248 \pm 14 \mathrm{~km} \mathrm{~s}^{-1}$ and its orbital period around the Galactic Centre $T=2 \pi /\left(\gamma \Omega_{0}\right)=$ $198 \pm 11 \mathrm{Myr}$, where the coefficient of dimensionality $\gamma=$ $1.023 \times 10^{-9}\left[\mathrm{~km} \mathrm{~s}^{-1} \mathrm{kpc}^{-1}\right] /\left[\right.$ radian $\left.\mathrm{yr}^{-1}\right]$ (Murray, 1986).


Figure 3. (a) Rotational angular velocity and (b) rotational circular linear velocity of the Galaxy according to (17) versus galactocentric distance. Dashed lines denote the bounds of the confidence interval corresponding to the $1 \sigma$ level. Vertical line indicates the location of $R_{0}=8 \mathrm{kpc}$.

### 3.4 Fourier Analysis of Velocities

At the next step, we consider deviations from circular velocities $\Delta V_{\theta}$ and galactocentric radial velocities $V_{R}$.

Spectral analysis consists in applying direct Fourier transform to the sequence of residual velocities $\Delta V$ in the following way:

$$
\begin{equation*}
\overline{\Delta V}\left(\lambda_{k}\right)=\frac{1}{M} \sum_{i}^{M} \Delta V^{i} \exp \left(-j \frac{2 \pi}{\lambda_{k}} R_{i}\right) \tag{13}
\end{equation*}
$$

where $\overline{\Delta V}\left(\lambda_{k}\right)$ is the $k$-th harmonic of Fourier transform, $M$ is the number of measurements of velocities $\Delta V^{i}$ with coordinates $R_{i}, i=1,2, \ldots, M$, and $\lambda_{k}$ is wavelength in kpc; the latter is equal to $D / k$, where $D$ is the period of the original sequence in kpc .

In density wave theory, $\lambda$ denotes the distance between adjacent spiral arms along the Galactic radius vector.

Understanding the results of spectral analysis is easy in the framework of linear theory of density waves. According to this approach (Burton \& Bania, 1974; Byl \& Ovenden, 1978),

$$
\begin{gather*}
\sigma_{1}=\widehat{\sigma} \cos \chi  \tag{14}\\
V_{R}=-f_{R} \cos \chi  \tag{15}\\
\Delta V_{\theta}=f_{\theta} \sin \chi \tag{16}
\end{gather*}
$$



Figure 4. Deviations from circular velocities $\Delta V_{\theta}$ (a) after subtraction of flat rotation curve from maser data and (b) after subtraction of the model rotation curve with parameters (17) versus galactocentric distance; vertical line indicates $R_{0}=8 \mathrm{kpc}$.


Figure 5. Power spectra of (a) deviations from circular velocities $\Delta V_{\theta}$ and of (b) radial velocities $V_{R}$.


Figure 6. Radial velocities of masers $V_{R}$ versus galactocentric distance $R$. Thin curves mark the confidence interval corresponding to $1 \sigma$ level. Vertical line indicates the galactocentric distance of the Sun $R_{0}=8 \mathrm{kpc}$.

## According Reid et al. (2009), Perseus spiral arm

has a pitch angle of 16+-3 deg.


Figure 7. Masers in the region of the Perseus arm in the $X Y$ Galactic plane. Velocity vectors with components $\left(V_{R}, \Delta V_{\theta}\right)$ are given for each maser. Location of the Sun is marked by circle.


Figure 7. Heliocentric spatial velocities $U^{1}$ and $V^{1}$ free from Galactic rotation with parameters (17).

All 28 masers give:
(U,V,W) =
$(-8.6,-13.6,-7.5)+-(2.1,1.6,1.3) \mathrm{km} / \mathrm{s}$.
And dispersions are:
$(\sigma U, \sigma V, \sigma W)=(10.8,8.7,6.8) \mathrm{km} / \mathrm{s}$.

Using 25 masers we found:
(U,V,W) =
$(-9.1,-15.5,-8.5)+-(1.9,1.3,1.2) \mathrm{km} / \mathrm{s}$.
And dispersions are:
$(\sigma U, \sigma V, \sigma W)=(9.3,6.6,6.1) \mathrm{km} / \mathrm{s}$.

## YOUNG OPEN STAR CLUSTERS




Figure 9. Power spectra of deviations from (a) circular velocities $\Delta V_{\theta}$ and (b) radial velocities $V_{R}$ for 117 young OSCs.

Figure 8. Deviations from (a) circular velocities $\Delta V_{\theta}$ and (b) radial velocities $V_{R}$ for 117 young OSCs versus galactocentric distance $R$. Approximation curves are shown with solid lines. Vertical line indicates galactocentric distance of the Sun $R_{0}=8 \mathrm{kpc}$.

For the local case, i.e. for the immediate Solar neighborhood, under the condition $\chi \approx \chi_{\odot}$ we have

$$
\begin{align*}
\bar{U}^{1} & =-\left(U_{\odot}\right)_{\mathrm{LSR}}-V_{R} \cos \theta+\Delta V_{\theta} \sin \theta,  \tag{18}\\
\bar{V}^{1} & =-\left(V_{\odot}\right)_{\mathrm{LSR}}+V_{R} \sin \theta+\Delta V_{\theta} \cos \theta . \tag{19}
\end{align*}
$$

When $i \rightarrow 0, \theta \rightarrow 0\left(R \approx R_{0}\right)$, and we have $\sin \theta \rightarrow 0$ and $\cos \theta \rightarrow 1$. Then

$$
\begin{align*}
\bar{U}^{1} & =-\left(U_{\odot}\right)_{\mathrm{LSR}}+f_{R} \cos \chi_{\odot},  \tag{20}\\
\bar{V}^{1} & =-\left(V_{\odot}\right)_{\mathrm{LSR}}+f_{\theta} \sin \chi_{\odot}, \tag{21}
\end{align*}
$$

where $f_{R}, f_{\theta}>0$. As one can see from (20)-(21), signs of corrections caused by spiral density waves are determined only by signs of $\cos \chi_{\odot}$ and $\sin \chi_{\odot}$. Adopting that: mean amplitudes of spiral density wave perturbations in both tangential and radial velocities are about $4-5 \mathrm{~km} \mathrm{~s}^{-1}$ and $\chi_{\odot}=-0.66 \pi$, we obtain, according to (20)-(21), the following "non-perturbed" velocity of $\left(U_{\odot}, V_{\odot}, W_{\odot}\right)_{\text {LSR }}=$ $(7,12,7) \pm(2,2,1) \mathrm{km} \mathrm{s}^{-1}$. This result agrees with the estimate SDB09 and FA09 Solar motion.

## ANOTHER APPROACH

## Bottlinger's equations:

$$
\begin{aligned}
& V_{r}=-U_{0} \cos (b) \cos \left(l-l_{0}\right)-V_{0} \cos (b) \sin \left(l-l_{0}\right)-W_{0} \sin (b)- \\
& -R_{0}\left(R-R_{0}\right) \cos (b) \sin \left(l-l_{0}\right) \Omega_{0}^{\prime}-R_{0}\left(R-R_{0}\right)^{2} \cos (b) \sin \left(l-l_{0}\right) \Omega_{0}^{\prime \prime} / 2+ \\
& +r \cos ^{2}(b) K+ \\
& +\cos (b)\left(V_{\theta} \sin \left(l-l_{0}+\theta\right)-V_{R} \cos \left(l-l_{0}+\theta\right)\right), \\
& V_{l}=-U_{0} \sin (b) \cos \left(l-l_{0}\right)-V_{0} \cos \left(l-l_{0}\right)+ \\
& +\left(\left(R-R_{0}\right) r \cos (b)-R_{0}\left(R-R_{0}\right) \cos \left(l-l_{0}\right)\right) \Omega_{0}^{\prime}+\left(\left(R-R_{0}\right) r \cos (b)-R_{0}\left(R-R_{0}\right)^{2} \cos \left(l-l_{0}\right)\right) \Omega_{0}^{\prime \prime} / 2+ \\
& +r \cos (b) \Omega_{0}+\left(V_{\theta} \cos \left(l-l_{0}+\theta\right)+V_{R} \sin \left(l-l_{0}+\theta\right)\right), \\
& V_{b}=U_{0} \sin (b) \cos \left(l-l_{0}\right)+V_{0} \sin (b) \sin \left(l-l_{0}\right)-W_{0} \cos (b)+ \\
& +R_{0}\left(R-R_{0}\right) \sin (b) \sin \left(l-l_{0}\right) \Omega_{0}^{\prime}+R_{0}\left(R-R_{0}\right)^{2} \sin (b) \sin \left(l-l_{0}\right) \Omega_{0}^{\prime \prime} / 2- \\
& -r \cos (b) \sin (b) K- \\
& -\sin (b)\left(V_{\theta} \sin \left(l-l_{0}+\theta\right)-V_{R} \cos \left(l-l_{0}+\theta\right)\right) .
\end{aligned}
$$

## According to (Lin,Yuan \& Shu, 1969):

$$
\begin{aligned}
& V_{R}=f_{R} \cos (\chi), \quad V_{\boldsymbol{\theta}}=f_{\boldsymbol{\theta}} \sin (\chi), \\
& \chi=m\left[\cot (i) \ln \left(R / R_{0}\right)-\theta\right]+\chi_{0} .
\end{aligned}
$$

## Nonlinear optimization problem:

We adopted Ro=8kpc, $\mathrm{m}=2, \mathrm{P}=12$ unknown parameters:

$$
\Omega_{0}, \Omega_{0}^{\prime}, \Omega_{0}^{\prime \prime}, U_{0}, V_{0}, W_{0}, l_{0}, K, f_{R}, f_{\theta}, i, \chi_{0}
$$

were found by solving the following nonlinear optimization problem:
min

$$
\begin{aligned}
& \delta^{2}=\frac{1}{(3 N-p)} \sum_{i=1}^{N} w_{r}^{i}\left(V_{r}^{i}-\hat{V}_{r}^{i}\right)^{2}+w_{l}^{i}\left(V_{l}^{i}-\hat{V}_{l}^{i}\right)^{2}+w_{b}^{i}\left(V_{b}^{i}-\hat{V}_{b}^{i}\right)^{2}, \\
& \text { where } S_{0} \text { - cosmic error, } N \quad \text { - number of data, } \\
& w_{r}=S_{0} / \sqrt{\left(S_{0}^{2}+\sigma_{v_{r}}^{2}\right.}, \quad w_{l}=\beta^{2} S_{0} / \sqrt{\left(S_{0}^{2}+\sigma_{v_{l}}^{2}\right.}, \quad w_{b}=\gamma^{2} S_{0} / \sqrt{\left(S_{0}^{2}+\sigma_{v_{b}}^{2}\right.} \quad S_{0}=8, \\
& \beta=\sigma_{V_{r}} / \sigma_{V_{l}}=1, \quad \gamma=\sigma_{V_{r}} / \sigma_{V_{b}}=2, \\
& \sigma_{V_{l}, V_{b}}=\frac{4.74}{\pi} \sqrt{\mu_{l, b}^{2}\left(\frac{\sigma_{\pi}}{\pi}\right)^{2}+\mu_{l, b}^{2} .}
\end{aligned}
$$

## RESULTS

Masers, $\mathrm{n}=\mathbf{2 8}$

| U0 | $10.4 \pm 1.7$ | $9.0 \pm 1.0$ |
| :--- | :---: | ---: |
| V0 | $9.9 \pm 2.0$ | $11.5 \pm 0.5$ |
| W0 | $7.4 \pm 0.6$ | $9.5 \pm 0.5$ |
|  |  |  |
| $\Omega 0$ | $-32.9 \pm 1.5$ | $-27.9 \pm 0.6$ |
| $\Omega 0$ | $5.1 \pm 0.2$ | $4.0 \pm 0.1$ |
| $\Omega 0^{\prime \prime}$ | $-1.04 \pm 0.06$ | $-0.62 \pm 0.20$ |
|  |  |  |
| K | $-2.9 \pm 0.7$ | $-1.1 \pm 0.2$ |
|  |  |  |
| fR | $-14.1 \pm 4.6$ | $-5.5 \pm 1.0$ |
| fӨ | $2.3 \pm 3.0$ | $3.0 \pm 1.0$ |
| X0 | $-180 \pm 60$ | $-120 \pm 20$ |
| i | $-4.7 \pm 1.5$ | $-5.0 \pm 0.2$ |

## CONCLUSIONS

Spatial velocities of 28 masers in 25 SFR having trigonometric parallaxes and located in the range of galactocentric distances $3<R<14 \mathrm{kpc}$ are analyzed.

To determine the Galactic rotation parameters we used the first three terms of the Taylor expansion of the angular rotation velocity at the galactocentric distance of the Sun Ro=8.0 kpc.

Fourier analysis of galactocentric radial velocities $V(R)$ allowed us to estimate amplitude $6.5 \pm 1 \mathrm{~km} / \mathrm{s}$ and wavelength $2 \pm 0.2 \mathrm{kpc}$ of the density wave periodic perturbations, and phase of the Sun in the density wave ( $-0.75 \pi$ ) - ( $-0.6 \pi$ ), what proofs that the Sun is located in the inter-arm space close to the Carina-Sagittarius arm.

We revised the localization of the Perseus spiral arm and found its pitch angle equal to $-5 \pm 1$ deg.

We obtained also components of the peculiar solar velocity with respect to the Local Standard of Rest, which are "non-perturbed" by the spiral density wave: (U,V,W)LSR = $(7,12,7)+-(2,2,1) \mathrm{km} / \mathrm{s}$.

